

# DESIGN CONCEPT FOR MICROWAVE RECURSIVE AND TRANSVERSAL FILTERS USING LANGE COUPLERS

IF2 I-7

L. BILLONNET - B. JARRY - P. GUILLON (Senior member IEEE)

I.R.C.O.M. - U.A. 356 C.N.R.S. - Université de Limoges  
123 avenue Albert Thomas - 87060 LIMOGES Cédex - FRANCE

## ABSTRACT

This study focuses on the use of Lange couplers in the design of passive recursive and transversal filters in the microwave frequency range. First, a simple numerical method for the computation of the filter parameters is described. A synthesis example is given to illustrate the techniques for the wideband filter case. We then show how Lange couplers can be effectively employed in the design of recursive and transversal filters. We present analytical, computer-simulated and experimental results for two passive band-pass filters which are implemented on a Duroïd substrate ( $\epsilon_r=2,43$  ;  $h=500$   $\mu\text{m}$ ) in the 3-5 GHz frequency range. Good agreement is obtained between the theoretical and the measured S-parameters for the filters.

## INTRODUCTION

Recent realization of recursive and transversal filters have shown that a given signal flowgraph may be translated into any number of physical designs [1][2][3]. Until now, such filters generally have been designed using a distributed circuit approach [4] employing active devices to compensate for losses and to improve the overall frequency response. As compared with these recently published papers, our methodology is oriented toward the filter parameters, with no deviation from the strict interpretations of low frequency principles. In the first part of this work, we describe, a novel simple matrix-based method for calculating the filter coefficients, to accurately fit an arbitrary transfer function. The method is applied to a wideband filter. The next part of our paper describes a new general circuit topology, which we analytically demonstrate to be capable of providing an arbitrary transversal or recursive filter response. We demonstrate the concept by mean of the design of two filters, one transversal and the other recursive. Measured S-parameters of these circuits show excellent agreement with theoretical analysis.

## THEORY

The designed response of such filters is achieved by combining signal components of each branch, with different amplitudes ( $a_k, b_p$ ) and frequency-dependent phase delays ( $\tau$ ).

Recursive and transversal filters are governed by the following time-domain equation

$$y(t) = - \sum_{p=1}^P b_p y(t-p\tau) + \sum_{k=0}^N a_k x(t-k\tau)$$

where  $x(t)[y(t)]$  is the input[output] of the system.

In the frequency domain, the corresponding equation becomes :

$$H(f) = \frac{Y(f)}{X(f)} = \frac{\sum_{k=0}^N a_k e^{-2j\pi f k \tau}}{1 + \sum_{p=1}^P b_p e^{-2j\pi f p \tau}}$$

We set  $f_0=1/\tau$  ; and then  $H(f)$  is periodic with period  $f_0$ . Consequently,  $H(f)$  can be put into the form :

$$H(f) = H_0(f) * \sum_{i=-\infty}^{+\infty} \delta(f - if_0)$$

We call  $H_0(f)$  the pattern of the transfer function  $H(f)$ ;  $f_0$  is equivalent to the width of  $H(f)$  pattern.

## NUMERICAL SYNTHESIS METHOD

Our method, derived from that presented by healy [3] for the transversal filter case, applies to the general recursive filter from as well. We write the transfer function  $H(f)$  at  $M$  uniformly spaced frequency points in the interval  $[0, f_0]$ , so that :

$$f_n = n \frac{f_0}{M-1} ; n \text{ varying in } [0, M-1]$$

$$H(n) = \frac{\sum_{k=0}^N a_k e^{-2j\pi n k / (M-1)}}{1 + \sum_{p=1}^P b_p e^{-2j\pi n p / (M-1)}} \quad (1)$$

With  $W = e^{-2j\pi/(M-1)}$ , (1) gives :

$$H(n) = \sum_{k=0}^N a_k W^{nk} - \sum_{p=1}^P b_p W^{np} H(n)$$

The unknown parameters of the system are  $\{a_k\}$  and  $\{b_p\}$ . We set  $I=N+P+1$ , the number of unknown coefficients so that  $I>M$ . With  $n$  varying from 0 to  $M-1$ , the above equation leads to the following over-determined matrix system.

$$\overrightarrow{H(n)} = \phi(n) \cdot \overrightarrow{A}$$

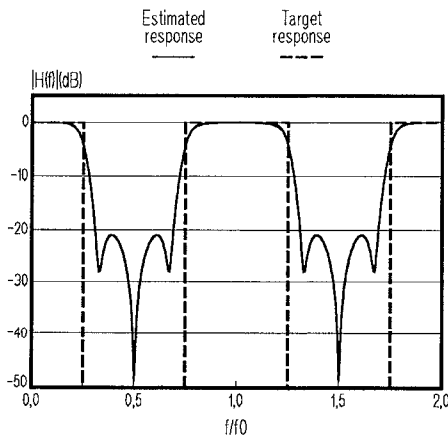
where  $\overrightarrow{H(n)} = [H(0), H(1), \dots, H(M-1)]^T$   
 $\overrightarrow{A} = [a_0, a_1, \dots, a_N, b_1, \dots, b_P]^T$   
 $\phi(n)$  is a  $N \times I$  matrix

From the least-squares error method, we have to solve the following matrix-system.

$$\phi(n)^T \phi(n) \cdot \overrightarrow{A} = \phi(n)^T \cdot \overrightarrow{H(n)}$$

where  $\phi(n)^T \phi(n)$  is a square matrix  $I \times I$   
 $\phi(n)^T \cdot \overrightarrow{H(n)}$  is a column matrix  $I \times 1$   
 $\overrightarrow{A}$  is the vector of unknown coefficients

A synthesis example, using our methodology for a wideband filter of order ( $N=3$ ;  $P=2$ ) is given in figure 1. This example successfully illustrates that the target frequency response can be reached for a relatively low order filter and for arbitrary transfer function shape.



- Figure 1 -

## POWER DIVIDERS

Recursive and transversal filters can turn to advantage of using Lange couplers. Indeed, couplers provides

great flexibility in matching input and output ports, and more generally, flexibility in the filter design. This approach enables filter branches to be easily associated and then to be designed separately from one to another. Moreover, the transposition of low frequency principles to microwaves domain is possible, especially for stability concepts. In keeping with this approach, we now discuss more precisely about Lange couplers, and about two filters, one transversal, and the other recursive, employing topologies based upon the use of such couplers.

## TRANSVERSAL APPROACH

Transversal filter model derives from general filter model by setting :

$$b_p = 0 ; p > 0$$

### Physical analysis approach using Lange couplers

In this paragraph, we consider a first order transversal filter governed by the following time-domain equation :

$$y(t) = a_0 x(t) + a_1 x(t-\tau)$$

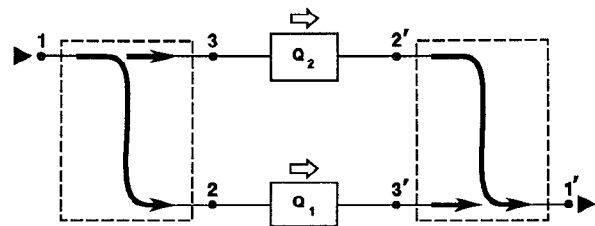
Corresponding transfer function is then given by :

$$S_{21}(f) = a_0 + a_1 e^{-2j\pi f\tau}$$

We consider the ideal coupler, with isolated port 4 loaded by a  $50\Omega$  impedance and where : 1 is the input port ; 2 is the direct output port ; 3 is the coupled output port. The S-matrix of the 3-ports device is :

$$(S) = e^{-j\omega\theta} * \begin{pmatrix} 0 & \alpha & j\beta \\ \alpha & 0 & 0 \\ j\beta & 0 & 0 \end{pmatrix}$$

New topology using 3-ports components is presented in figure 2 where  $Q_1$  and  $Q_2$  are characterized by their S-matrix ( $S_1$ ) and ( $S_2$ ).



- Figure 2 -

Considering that :

$\alpha = \beta$  (3 dB Lange couplers)

$Q_1$  is a  $l_1$  length of  $50\Omega$  transmission line

$Q_2$  is a  $l_2$  length of  $50\Omega$  transmission line

with  $l_2 - l_1 = \lambda_0$ ,  $\lambda_0$  wavelength corresponding to  $f_0$

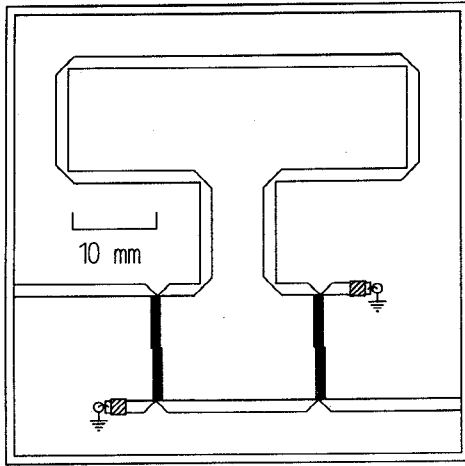
it can be shown that  $S_{11} = S_{22} = 0$ .

And  $S_{21}(f) = j e^{-4j\pi f\theta} (a_0 + a_1 e^{-2j\pi f\tau})$

where  $j e^{-4j\pi f\theta}$  is a term introduced by the couplers  
 $a_0 = a_1 = 1/2$  are the coefficients of the filter  
 $\tau = 1/f_0$

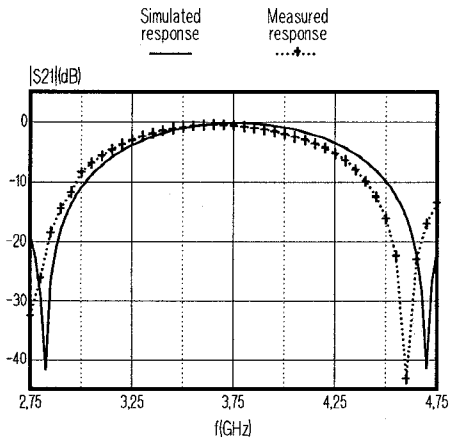
### Experimental approach

Our transversal realization is presented in figure 3.  
The circuit has been implemented on a 500  $\mu\text{m}$ -thick Duroid substrate ( $\epsilon_r=2,43$ ).



- Figure 3 -

The filter is designed to work in the 3-5 GHz range, implying the pattern of  $H(f)$  to be 2 GHz wide. Excellent agreement is obtained between computer-simulated results and measured S-parameters as shown in figure 4.



- Figure 4 -

### RECURSIVE APPROACH

Basic recursive filter model derives from general filter model by setting :

$$a_k = 0 ; k > 0$$

### Physical analysis approach using Lange couplers

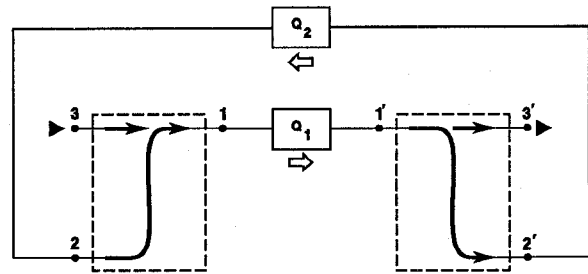
In this paragraph, we focus on a first order recursive filter governed by the following time-domain equation :

$$y(t) = a_0 x(t) - b_1 y(t-\tau)$$

Corresponding transfer function is then given by :

$$S_{21}(f) = \frac{a_0}{1 + b_1 e^{-2j\pi f\tau}}$$

Recursive topology using 3-ports devices is presented in figure 5.



- Figure 5 -

We set that :

$Q_1$  is a  $l_1$  length of  $50\Omega$  transmission line

$Q_2$  is a  $l_2$  length of  $50\Omega$  transmission line

With  $2\theta + \tau_1 + \tau_2 = \tau$ , the circuit S-parameters are :

$$S_{11} = S_{22} = 0$$

$$S_{21}(f) = -e^{-2j\pi(2\theta + \tau_1)} \frac{a_0}{1 + b_1 e^{-2j\pi f\tau}}$$

where  $-e^{-2j\pi(2\theta + \tau_1)}$  is introduced by the couplers and the  $50\Omega$  line  $Q_1$

$a_0 = \beta^2$  is the coefficient of the transversal part of the filter

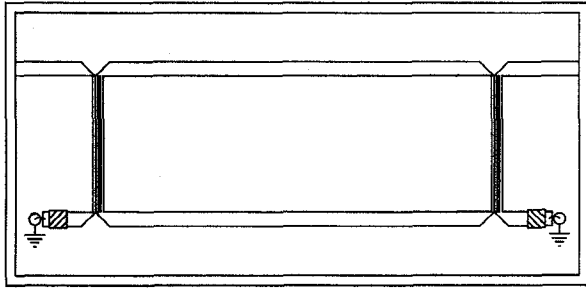
$b_1 = -\alpha^2$  is the coefficient of the recursive part of the filter

### Experimental approach

The circuit is presented in figure 6. Couplers were designed to obtain :

$$|S_{21}|_{\text{MAX}} / |S_{21}|_{\text{MIN}} \equiv |S_{21}(f_c)| / |S_{21}(f_c - f_0/2)| = 10$$

We find that  $\alpha^2 = 9/11$  and  $\beta^2 = 1 - \alpha^2 = 2/11$ . Coupling value  $K = 20 \log_{10}(\beta)$  is then set to -7.4 dB.

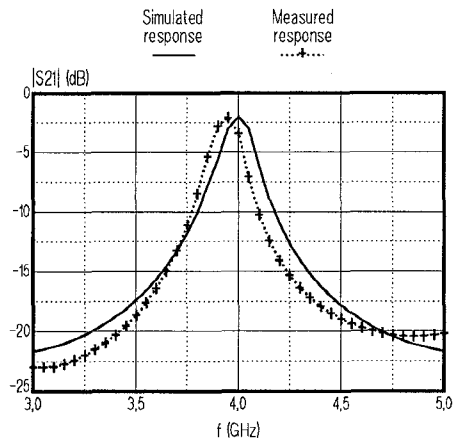


10 mm

- Figure 6 -

We note that couplers employed are "unfolded version" of classical Lange couplers. Indeed, in 1972, Waugh and Lacombe [5] demonstrated that the performance is the same as that of Lange couplers, thus providing flexibility in geometrical layout.

Excellent agreement is shown in figure 8 between computer-simulated results and measured S-parameters.



- Figure 7 -

## CONCLUSION

The main purpose of this paper is to analyse the methodology of using Lange couplers in recursive and transversal filters designs, in keeping with the strict interpretations of low frequency principles. The design approach has been outlined for both transversal and recursive type circuits. By using Lange couplers with various coupling values, and then calculating the corresponding filter parameters, we have readily compared theoretical results with computer-simulated ones. Experimental filter examples, comprising one of each kind, further illustrates the validity of our approach.

## Acknowledgements :

This work is supported by the European Space Agency under contract n°7808/88/NL/JG for a non agile very narrow band channel filters applications.

## REFERENCES

- [1] S.E. SUSSMAN-FORT  
"Design concepts for microwave GaAs FET active filters"  
*IEEE Trans. on MTT*, vol.MTT-37, Sept. 1989, pp.1418-1424
- [2] C. RAUSCHER  
"Microwave active filters based on transversal and recursive principles"  
*IEEE Trans. on MTT*, vol.MTT-33, Dec. 1985, pp.1350-1360
- [3] M. HEALY and al.  
"Active filters of MMIC"  
*17e European Microwave Conference Proceedings*, Sept. 1987, Rome, Italy
- [4] M.J. SCHINDLER and Y. TAJIMA  
"A novel MMIC active filter with lumped and transversal elements"  
*IEEE Trans. on MTT*, vol.MTT-37, Dec. 1989, pp.2148-2153